

Higgs mass determined by cosmological parameters

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Postulating that all massless elementary fields have conformal scaling symmetry removes a conflict between gravitational theory and the standard model of elementary quantum fields. If the scalar field essential to SU(2) symmetry breaking has conformal symmetry, it must depend explicitly on the Ricci curvature scalar of gravitational theory. This has profound consequences for both cosmology and elementary particle physics, since cosmological data determine scalar field parameters. A modified Friedmann equation is derived and solved numerically. The theory is consistent with all relevant data for supernovae redshifts below $z = 1$. The implied value of the cosmological constant implies extremely small Higgs mass, far below current empirical lower bounds. Detection of a Higgs boson with large mass would falsify this argument.

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I. INTRODUCTION

Einstein gravitational theory has lower symmetry than the standard model of spinor and gauge boson fields. Massless electroweak action integrals are invariant under local Weyl (conformal) scaling, $g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x)e^{2\alpha(x)}$, for fields with definite conformal character. For example, scalar field $\Phi(x) \rightarrow \Phi(x)e^{-\alpha(x)}$. A conformal energy-momentum 4-tensor is traceless, while the Einstein tensor is not. Compatibility can be imposed by replacing the Einstein-Hilbert field action by a uniquely determined action integral constructed using the conformal Weyl tensor, which preserves general relativistic phenomenology at the distance scale of the solar system[1]. Discrepancies at galactic distances are commonly attributed to unobserved dark matter. Most remarkably, conformal theory is consistent with the empirically successful MOND model[2] in describing the systematics of these discrepancies. This provides an alternative explanation of excessive rotational velocities in the outer regions of galaxies, without invoking dark matter[1].

In the electroweak standard model[3], mass is generated by an SU(2) doublet complex scalar field Φ . The Lagrangian density contains $\Delta\mathcal{L}_\Phi = w^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$, where w^2 and λ are positive constants. The Higgs mass is $m_H = \sqrt{2}w$. Units here are such that $\hbar = c = 1$. $\lambda(\Phi^\dagger\Phi)^2$ is conformally covariant, but conformal symmetry is broken by $w^2\Phi^\dagger\Phi$. The Higgs construction breaks SU(2) and conformal symmetries by setting $\Phi^\dagger\Phi = \phi_0^2$, for spacetime constant ϕ_0 . Conformal theory replaces the w^2 term by $-\frac{1}{6}R\Phi^\dagger\Phi$, where scalar $R = g_{\mu\nu}R^{\mu\nu}$ for Ricci tensor $R^{\mu\nu}$, the symmetric contraction of the gravitational Riemann tensor[1]. However, empirical cosmological parameters indicate that $R > 0$. This sign conflict is resolved here by including both terms, while ϕ_0 is modified to include the effect of $w^2 - \frac{1}{6}R$. The residual constant term in the Lagrangian density defines an effective cosmological constant $\bar{\Lambda}$ [1].

If w^2 is a dynamical result of self-interaction [4], it may be much smaller than commonly assumed. Conformal covariance of the bare electroweak and gravitational theories establishes a relationship between parameters w^2 and $\bar{\Lambda}$. The latter is currently known from empirical cosmology, implying a very small value of w^2 and Higgs mass m_H many orders of magnitude smaller than empirical limits inferred using the standard model[5]. However, important mechanisms of Higgs production and detection would be removed if Higgs-fermion coupling were eliminated[4], which would require reconsideration of theoretical expectations. In any case, detecting a spinless neutral particle with mass less than the electron neutrino presents a great experimental challenge.

II. CONFORMAL THEORY INCLUDING A SCALAR FIELD

For homogeneous isotropic (Robertson-Walker) geometry, the conformal gravitational term vanishes in the coupled field equations, requiring the total source energy-momentum tensor to vanish[1]. However, the Ricci scalar in the Lagrangian density of a conformal scalar field produces a gravitational term in its energy-momentum tensor. This implies an effective gravitational field equation. Scalar field parameters determine a cosmological constant (aka dark energy) in the resulting cosmological Friedmann equation. Empirical values of the relevant parameters can be fitted to redshift data implying Hubble expansion and acceleration[1]. Thus in conformal theory the scalar field, postulated to generate gauge boson mass in the standard model, is also responsible for cosmological expansion. Qualitatively, this is not surprising, since the Higgs mechanism renormalizes the universal vacuum state, producing a nonvanishing scalar field amplitude ϕ_0 throughout the universe. Its energy-momentum tensor is a cosmological entity.

Following sign conventions of electroweak theory[3, 6], diagonal metric tensor $g_{\mu\nu}$ has elements $(1, -1, -1, -1)$ in flat space. A covariant energy-momentum tensor

$$\Theta_a^{\mu\nu} = \frac{-2\delta I_a}{\sqrt{-g}\delta g_{\mu\nu}}. \quad (1)$$

is determined by any invariant action integral $I_a = \int d^4x \sqrt{-g} \mathcal{L}_a$. This defines Θ_a^{00} as an energy density. Conformal symmetry implies that $\Theta_a^{\mu\nu}$ is traceless. For gravitational Lagrangian density \mathcal{L}_g , functional derivative

$$X_g^{\mu\nu} = \frac{\delta I_g}{\sqrt{-g}\delta g_{\mu\nu}} \quad (2)$$

implies the gravitational field equation

$$X_g^{\mu\nu} = \frac{1}{2} \sum_a \Theta_a^{\mu\nu}. \quad (3)$$

If $\delta \mathcal{L}_g = x_g^{\mu\nu} \delta g_{\mu\nu}$, up to a 4-divergence, the functional derivative of action integral I_g is $X_g^{\mu\nu} = x_g^{\mu\nu} + \frac{1}{2} \mathcal{L}_g g^{\mu\nu}$. Standard Einstein-Hilbert theory, with Ricci scalar $R = g_{\mu\nu} R^{\mu\nu}$, cosmological constant Λ , and $\mathcal{L}_g = (R - 2\Lambda)/2\kappa$, implies the Einstein field equation

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = -\kappa \sum_a \Theta_a^{\mu\nu}, \quad (4)$$

where $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}$ is the Einstein tensor. Its trace $G = -R$ does not in general vanish.

Uniform, isotropic cosmology is characterized by Robertson-Walker (R-W) geometry. With the present sign conventions, the metric tensor is defined for spatial curvature k by

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \quad (5)$$

The Ricci tensor $R^{\mu\nu}$ (contracted Riemann) depends on universal scale factor $a(t)$ through only two independent functions, $\xi_0(t) = \frac{\ddot{a}}{a}$ and $\xi_1(t) = \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}$. The Ricci scalar is $R(t) = 6(\xi_0(t) + \xi_1(t))$. In R-W geometry, functional derivative $X_g^{\mu\nu}$ vanishes for the conformal gravitational Lagrangian[1]. If averaged uniform matter and radiation produce $\Theta_m^{\mu\nu}$, the field equation reduces to $\Theta_\Phi^{\mu\nu} + \Theta_m^{\mu\nu} = 0$.

Electroweak theory[6, 7] postulates an SU(2) doublet complex scalar field Φ whose Lagrangian density contains

$$\Delta \mathcal{L}_\Phi = w^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2, \quad (6)$$

omitting a constant term. If ϕ_0 is a spacetime constant, and $\phi_0^2 = \frac{w^2}{2\lambda}$ for $\lambda > 0$, $\Phi = \phi_0$ is an exact global solution of the scalar field equation. This determines a stable vacuum state. $\Delta \mathcal{L}_\Phi$ has residual value $\frac{1}{2} w^2 \phi_0^2$.

$\mathcal{L}_\Phi = (\partial_\mu \Phi)^\dagger \partial^\mu \Phi - \frac{1}{6} R \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$ defines a conformally invariant action integral[1]. Electroweak theory adds a term $w^2 \Phi^\dagger \Phi$, which breaks conformal symmetry. In R-W geometry, the occurrence of Ricci scalar R here

implies a Friedmann cosmic evolution equation with coefficients determined by the scalar field Lagrangian[1]. R varies on a cosmological time scale. The resulting time variation of ϕ_0 is many orders of magnitude smaller than that relevant to elementary particle physics. The present analysis will treat R as a constant in the scalar field equation, $\partial_\mu \partial^\mu \Phi = (-\frac{1}{6} R + w^2 - 2\lambda \Phi^\dagger \Phi) \Phi$.

If constant ϕ_0 , such that $\phi_0^2 = w^2/2\lambda$, is substituted for the scalar field, as in the Higgs construction, $\mathcal{L}_\Phi = -\frac{1}{6} \phi_0^2 (R - 3w^2)$. This acts as an effective gravitational Lagrangian density, of the same form as Einstein-Hilbert, but with parameters $\bar{\kappa} = -\frac{3}{\phi_0^2}$, $\bar{\Lambda} = \frac{3}{2} w^2$. The implied field equation is $G^{\mu\nu} + \bar{\Lambda} g^{\mu\nu} = -\bar{\kappa} \Theta_m^{\mu\nu}$. The gravitational constant is negative and the cosmological constant is determined by scalar field parameters[1].

Generalizing the Higgs construction, for ϕ_0 such that $\phi_0^2 = \frac{1}{2\lambda} (w^2 - \frac{1}{6} R)$, $\Phi = \phi_0$ is a global solution of the scalar field equation. The time derivative of R can be neglected in the scalar field equation. The algebraic sign of λ must agree with $w^2 - \frac{1}{6} R$. Because \mathcal{L}_Φ depends on R , its functional derivative for variation of the metric tensor is $X_g^{\mu\nu} = \frac{1}{6} R^{\mu\nu} \Phi^\dagger \Phi + \frac{1}{2} \mathcal{L}_\Phi g^{\mu\nu}$. Evaluated for $\phi_0^2 = \frac{1}{2\lambda} (w^2 - \frac{1}{6} R)$, $\mathcal{L}_\Phi = \phi_0^2 (w^2 - \frac{1}{6} R - \lambda \phi_0^2) = \frac{1}{6} \phi_0^2 (3w^2 - \frac{1}{2} R) = \frac{1}{4\lambda} (w^2 - \frac{1}{6} R)^2$. Hence the effective gravitational functional derivative is $X_g^{\mu\nu} = \frac{1}{6} \phi_0^2 (R^{\mu\nu} - \frac{1}{4} R g^{\mu\nu} + \frac{3}{2} w^2 g^{\mu\nu})$. The middle term here is reduced by a factor of two from the corresponding Einstein-Hilbert expression. Defining $\bar{\kappa} = -\frac{3}{\phi_0^2}$ and $\bar{\Lambda} = \frac{3}{2} w^2$, the modified gravitational field equation is $R^{\mu\nu} - \frac{1}{4} R g^{\mu\nu} + \bar{\Lambda} g^{\mu\nu} = -\bar{\kappa} \Theta_m^{\mu\nu}$.

III. THE COSMOLOGICAL FRIEDMANN EQUATION

In the R-W metric, $R^{00} = 3\xi_0$ and $R = 6(\xi_0 + \xi_1)$. For energy density $\Theta_m^{00} = \rho$, the effective Einstein field equation implies standard Friedmann cosmic evolution equation $-\frac{1}{3} G^{00} = \xi_1(t) = \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{3} (\bar{\kappa} \rho + \bar{\Lambda})$, expressed in terms of scalar field parameters. The effective gravitational field equation derived above, taking into account Lagrangian term $-\frac{1}{6} R \Phi^\dagger \Phi$ and the modified Higgs construction, implies $R^{00} - \frac{1}{4} R = 3\xi_0 - \frac{3}{2} (\xi_0 + \xi_1) = -\bar{\kappa} \rho - \bar{\Lambda}$. This reduces to a modified Friedmann equation

$$\xi_1(t) - \xi_0(t) = \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\ddot{a}}{a} = \frac{2}{3} (\bar{\kappa} \rho + \bar{\Lambda}). \quad (7)$$

For positive energy density, $\bar{\kappa} \rho$ here is negative, compensated by $\bar{\Lambda}$, spatial curvature k , and acceleration \ddot{a} .

In the early universe, with no stable masses, conformal symmetry requires trace $\Theta_m = g_{\mu\nu} \Theta_m^{\mu\nu}$ to vanish. Trace Θ_Φ must also vanish. Using $g_{\mu\nu} (R^{\mu\nu} - \frac{1}{4} R g^{\mu\nu}) = 0$, gravitational trace terms cancel identically in the effective field equation, implying $\bar{\Lambda} = 0$. Then for $k \rightarrow 0$ Eq.(7) reduces to $\frac{d}{dt} \frac{\dot{a}}{a} = -\frac{2}{3} \bar{\kappa} \rho$, which correctly implies exponential expansion because $\bar{\kappa} < 0$ in conformal theory. This is consistent with the postulate of universal

conformal symmetry[1], prior to dynamical symmetry-breaking, and with the hypothesis that parameter w^2 is due to self-interaction associated with such symmetry-breaking[4]. For temperatures below the electroweak transition temperature T_{EW} , conformal symmetry is broken dynamically by the Higgs mechanism. In this epoch, trace Θ_m cannot be assumed to vanish. Once conformal symmetry is broken and stable nonzero mass is possible, R-W geometry is modified by mass concentrations and may not remain strictly valid as a cosmological model.

IV. EMPIRICAL PARAMETERS

In the present conformal theory scalar field parameters w^2, λ determine scalar amplitude ϕ_0 , cosmological constant $\bar{\Lambda}$, and effective gravitational constant $\bar{\kappa}$. In a Robertson-Walker metric, scale factor $a(t)$ defines expansion rate $H(t) = \dot{a}(t)/a(t)$, whose present value $H_0 = H(t_0)$ is the Hubble constant. The parametrized gravitational equations determine a Friedmann cosmic evolution equation which relates $\bar{\Lambda}$ and energy density ρ to cosmic expansion $a(t)$ and acceleration $\frac{\ddot{a}a}{\dot{a}^2} = -q(t)$. Defining dimensionless quantities, and normalizing to $a(t_0) = 1$ at present time t_0 ,

$$\Omega_m = \frac{2\bar{\kappa}\rho}{3H_0^2}; \Omega_\Lambda = \frac{2\bar{\Lambda}}{3H_0^2} = \frac{w^2}{H_0^2}; \Omega_k = -\frac{k}{H_0^2}; \Omega_q = -q(t_0),$$

the modified Friedmann Eq.(7) reduces at t_0 to

$$\Omega_m + \Omega_\Lambda + \Omega_k + \Omega_q = 1. \quad (8)$$

Coefficient $\bar{\kappa}$ is unrelated to the Newton constant $8\pi G_N$, which retains its validity for gravitational dynamics in the solar system. The full conformal theory is required for nonuniform or nonisotropic energy-momentum density on a galactic scale[1]. The sign reversal of $\bar{\kappa}$ relative to G_N implies $\Omega_m \leq 0$. Empirical values of Ω_Λ are positive, while Ω_k is near zero. If $\bar{\Lambda} > 0$ and $k < 0$, the evolution equation implies that the current value of Ω_m is very small[1]. Given $0 < \Omega_\Lambda + \Omega_k < 1$, empirically well-defined, the residual parameter from the modified Friedmann equation is $1 > \Omega_m + \Omega_q > 0$, which is compatible with $\Omega_m \leq 0$. In contrast, the standard dimensionless Friedmann equation, $\Omega_m + \Omega_\Lambda + \Omega_k = 1$ implies $1 > \Omega_m > 0$, conflicting with conformal theory. Conclusions regarding Ω_m based on the standard equation may require reconsideration.

The algebraic sign of parameter λ must agree with $w^2 - \frac{1}{6}R$. At t_0 the independent field parameters are $\xi_0(t_0) = \Omega_q H_0^2$ and $\xi_1(t_0) = (1 - \Omega_k)H_0^2$, so that $\frac{1}{6}R = \xi_0 + \xi_1 = (1 - \Omega_k + \Omega_q)H_0^2$. Thus $(w^2 - \frac{1}{6}R)/H_0^2 = \Omega_\Lambda + \Omega_k - \Omega_q - 1 = -2\Omega_q - \Omega_m$, negative for $\Omega_m < 0$ if $\Omega_q + \Omega_m > 0$. A consistent model with $\Omega_q + \Omega_m > 0$ requires $\lambda < 0$. Because the resulting value of $\mathcal{L}_g = \frac{1}{4\lambda}(w^2 - \frac{1}{6}R)^2$ is negative, $\lambda < 0$ does not destabilize the physical vacuum ground state.

V. NUMERICAL SOLUTION OF THE MODIFIED FRIEDMANN EQUATION

The modified Friedmann equation $\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a} = \alpha - \frac{k}{a^2} - \frac{\beta}{a^3}$ can be solved numerically, given constant parameters $\alpha = \frac{2}{3}\bar{\Lambda} > 0$, $k \simeq 0$, and $\beta = -\frac{2}{3}\bar{\kappa}\rho a^3 > 0$. For luminosity distance d_L , $H_0 d_L$ is computed as a function of redshift z . By adjusting the free parameter Ω_q , Mannheim[8] fitted his Eq.(11), a solution of the standard Friedmann equation for $k = 0$ and $\beta = 0$, to empirical redshift data. The fitted function is indistinguishable from a standard Hubble plot of the same data, with consensus parameters $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$. Similar results are given here for the modified Friedmann equation, also with vanishing k and β . Here Ω_q is determined by the solution, while free parameter α is adjusted to match Mannheim's $H_0 d_L$ for redshift $z \simeq 1$. The functions agree to graphical accuracy for $z \leq 1$. Remarkably, the fitted parameter $\Omega_\Lambda = \alpha = 0.732$ for $\Omega_k = 0$ is consistent with current empirical values $\Omega_\Lambda = 0.726 \pm 0.015$, $\Omega_k = -0.005 \pm 0.013$ [9]. Any significant discrepancy would invalidate the present theory.

z	Ω_Λ	Ω_q	$H_0 d_L (calc)$	$H_0 d_L ([8])$
0.000	0.732	0.268	0.000	0.000
0.063	0.672	0.328	0.066	0.066
0.133	0.619	0.381	0.145	0.145
0.211	0.571	0.429	0.240	0.241
0.298	0.530	0.470	0.355	0.357
0.395	0.492	0.508	0.494	0.497
0.503	0.459	0.541	0.663	0.666
0.623	0.428	0.572	0.868	0.871
0.758	0.401	0.599	1.118	1.121
0.909	0.376	0.624	1.424	1.426
1.079	0.353	0.647	1.799	1.799

VI. RELATIONSHIP TO STANDARD ELECTROWEAK THEORY

Because the standard model Lagrangian for the $SU(2)$ complex scalar field omits Ricci scalar R , the implied scalar field equation has a time-independent global solution. If both w^2 and λ are positive, this implies Higgs symmetry-breaking. Conformal theory replaces w^2 by $w^2 - \frac{1}{6}R$, where R is time-dependent on a cosmological scale, as implied by the modified Friedmann equation. This extremely weak time-dependence establishes a correspondingly small but nonzero coupling to the Z^0 weak boson field, through the $SU(2)$ covariant derivative that acts on the scalar field[6, 7]. If virtual emission and absorption of the Z^0 field produces a dressed scalar field, parameters w^2 and λ would represent the implied induced self-interaction[4]. Because the Z^0 transition amplitudes depend on cosmological time-derivatives, parameter values would be extremely small.

The current empirical value of the Hubble constant is $H_0 \simeq 70.5 \text{ km/s/Mpc}$ [9]. Using $\Omega_\Lambda = 0.726$, the present theory implies Higgs mass $m_H = \sqrt{2\Omega_\Lambda} H_0 = 1.81 \times 10^{-33} \text{ eV}$. The standard Friedmann equation implies essentially the same result, differing by a factor of order unity.

VII. CONCLUSIONS

It has been shown here that requiring fundamental gravitational and scalar field action integrals to have conformal scaling symmetry, well-established for massless spinors coupled to gauge boson fields, provides plausible explanations of several puzzling phenomena in elementary particle and gravitational physics. The most striking are the long-term failure to observe a massive Higgs boson, and the need in empirical cosmology for dark energy or an equivalent cosmological constant [1].

Conformal theory justifies an effectively unified gravitational and elementary-particle theory. The postulated SU(2) scalar boson field is the common element that links these traditionally incompatible theories. There is no need to quantize gravitational theory or to geometrize

quantum field theory. A classical metric field serves as the blackboard upon which quantum theory is written.

Conformal theory relates gravitational and electroweak parameters through the cosmological constant $\bar{\Lambda} = \frac{3}{2}w^2$. The Higgs construction of electroweak theory, which assumes $w^2 > 0$, establishes a nonzero cosmological constant, equivalent to uniformly distributed dark energy. This parameter can be attributed to self-interaction of the scalar field, due to virtual excitation of the Z^0 gauge field [4]. This process is effective only in the current epoch of cosmic evolution, when mean temperature is below the electroweak transition temperature T_{EW} .

If this is the correct explanation of empirical dark energy, it implies an extremely small Higgs boson mass, found here to be $m_H \simeq 10^{-33} \text{ eV}$. Although this value might appear to be unreasonably small, no heavier Higgs boson has been detected. Arguments that exclude small Higgs mass [5] depend on Higgs-fermion coupling. As has recently been shown [4], a modified symmetry postulate in the standard model removes such coupling terms, while justifying a plausible estimate of neutrino mass.

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